On-demand Entanglement Source with Polarization-Dependent Frequency Shift

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We propose a polarization-dependent spatial phase modulation method to purify the two-photon polarization entanglement generated by the biexciton cascade decay in a single semiconductor quantum dot. In principle, our method can completely compensate the random phase acquired from the decay of the non-degenerate exciton states in time domain. In frequency domain, our method is equivalent to shifting photon frequency according to its polarization. The method can be applied immediately by existing experimental set-ups.

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Introduction.— Quantum entanglement plays an important role in the study of fundamental principles of quantum mechanics[1]. It is also the most important resource in quantum information processing[2, 3]. Among all types of quantum entanglement, polarization entangled photon-pairs are particularly useful because of easy manipulation and transmission. There are many matured techniques to produce such entangled pairs *probabilistically*[4, 5, 6, 7], while an *on-demand* entangled photon pair is essential in many tasks in quantum information processing.

Recently, an on-demand entangled photon-pair source was proposed[8] and realized in a semiconductor quantum dot system[9, 10, 11]. However, because of fine-structure splitting (FSS), the relative phase of the entangled state is randomized so that only classical correlation can be detected by traditional time-integrated measurement[12, 13]. So far, there are many methods proposed to explore this "hidden entanglement", for example, reducing FSS[14, 15, 16, 17], spectral filtering[10], time resolving post-selection[13], and so on. Up to now, the smallest FSS realized in experiment is about $0.3\,\mu eV$ and non-classical nature of the radiation field is verified by directly observing violation of the Bell inequality[17]. However, the entanglement quality is considerably decreased even by very small FSS, and further reducing FSS is very difficult in experiment. Furthermore, the severe restriction on FSS greatly limits the selection range of quantum dot systems. Certain quantum dots with large FSS cannot be used even if they have distinct advantage, such as emitting photons of frequencies in the easy transmission frequency window in free space or optical fiber. Also, the post-selection method in frequency domain or time domain will significantly decrease the photon collection efficiency.

Our method.— Here we propose an experimental scheme to get rid of *all* the drawbacks listed above. In principle, our method can *completely* compensate the random phase resulted from FSS, and therefore, greatly enlarge the selection range of quantum dot systems. Meanwhile, our proposal just slightly reduces the photon-pair collection rate due to the loss of the phase modulator employed. The key point in our method is polarization-dependent spatial phase modulation, which is equivalent to a polarization-dependent frequency shift operation $U(\Delta_1, \Delta_2)$ defined as

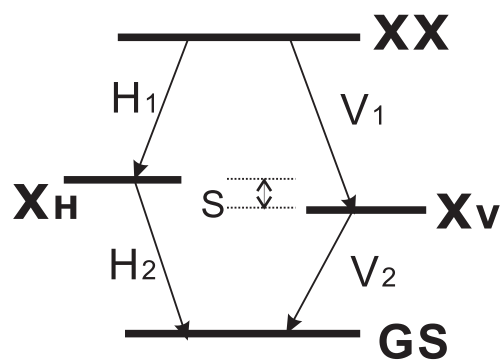

FIG. 1: Energy levels of the semiconductor quantum dot used to generate polarization entangled photons. The biexciton state (XX) is a zero-spin state formed by two electrons and two heavy holes. When the dot decays, two photons are emitted sequentially, and their polarization is determined by the "decay path". Usually an FSS S exists between the two excitons (X_H) and (X_V).

$$
\begin{aligned}
U(\Delta_1,\Delta_2)|H_1H_2;\omega_1,\omega_2\rangle &= |H_1H_2;\omega_1,\omega_2\rangle;\\
U(\Delta_1,\Delta_2)|V_1V_2;\omega_1,\omega_2\rangle &= |V_1V_2;\omega_1+\Delta_1,\omega_2+\Delta_2\rangle.
\end{aligned}
\tag{1}
$$

Here H and V stand for horizontal and vertical polarization, respectively, ω is the photon frequency, Δ is an arbitrary frequency shift, and the subscripts 1 and 2 refer to the first photon and second photon, respectively. After such a unitary transformation is applied, FSS can be completely compensated, and therefore, "hidden entanglement" revives.

The energy levels of the quantum dot used for photon-pair generation are shown in Fig. 1. After exciting a single quantum dot into biexciton state (XX), two photons are emitted sequentially as the dot decays in a cascade process. Because the two exciton states (X_H and X_V) are not degenerate[18, 19], the two photons are actually entangled in the complex space of both polarization and frequency

$$
\begin{aligned}
|\Psi\rangle = \frac{1}{\sqrt{2}}\Big[&\iint_{-\infty}^{\infty} d\omega_1 d\omega_2 \Phi_H(\omega_1,\omega_2)|H_1H_2;\omega_1,\omega_2\rangle\\
&+\iint_{-\infty}^{\infty} d\omega_1 d\omega_2 \Phi_V(\omega_1,\omega_2)|V_1V_2;\omega_1,\omega_2\rangle\Big],
\end{aligned}
\tag{2}
$$

After $U(\Delta_1, \Delta_2)$ is applied, the state of the two photons is changed into

$$\begin{aligned} & U(\Delta_1,\Delta_2)|\Psi\rangle \\ =& \frac{1}{\sqrt{2}}\Big[\iint_{-\infty}^{\infty} d\omega_1 d\omega_2 \Phi_H(\omega_1,\omega_2)|H_1H_2;\omega_1,\omega_2\rangle \\ & + \iint_{-\infty}^{\infty} d\omega_1 d\omega_2 \Phi_V(\omega_1-\Delta_1,\omega_2-\Delta_2)|V_1V_2;\omega_1,\omega_2\rangle\Big]. \end{aligned} \tag{3}$$

If the two spectral functions satisfy

$$\Phi_H(\omega_1,\omega_2) = \Phi_V(\omega_1-\Delta_1,\omega_2-\Delta_2), \tag{4}$$

the frequency space and polarization space are completely separated and the two photons become maximally entangled in polarization space:

$$U(\Delta_1,\Delta_2)|\Psi\rangle = |\Phi^{(+)}\rangle \otimes \iint_{-\infty}^{\infty} d\omega_1 d\omega_2 \Phi_H(\omega_1,\omega_2)|\omega_1,\omega_2\rangle, \tag{5}$$

where $|\Phi^{(+)}\rangle = \frac{1}{\sqrt{2}}(|H_1H_2\rangle + |V_1V_2\rangle)$.

The spectral functions for the two decay path of the quantum dot system can be written as[10, 20]

$$\begin{aligned} \Phi_H(\omega_1,\omega_2) =& \frac{\sqrt{2}\Gamma}{2\pi}\frac{1}{\omega_1+\omega_2-\omega_0+i\Gamma} \\ & \times \frac{1}{\omega_2-\omega_{H_2}+i\Gamma/2}, \end{aligned} \tag{6a}$$

$$\begin{aligned} \Phi_V(\omega_1,\omega_2) =& \frac{\sqrt{2}\Gamma}{2\pi}\frac{1}{\omega_1+\omega_2-\omega_0+i\Gamma} \\ & \times \frac{1}{\omega_2-\omega_{V_2}+i\Gamma/2}. \end{aligned} \tag{6b}$$

Here, as shown in Fig. 1, $\omega_{H_2} = \omega_{X_H} - \omega_{GS}$, $\omega_{V_2} = \omega_{X_V} - \omega_{GS}$, and $\omega_0 = \omega_{XX} - \omega_{GS}$, where $\hbar\omega_{XX}$, $\hbar\omega_{X_H}$, $\hbar\omega_{X_V}$, $\hbar\omega_{GS}$ are the eigenenergy of levels XX, X_H, X_V, and GS, respectively, and Γ is the decay rate of the four transitions $XX \to X_H$, $XX \to X_V$, $X_H \to GS$, and $X_V \to GS$[10]. By noting that

$$\begin{aligned} \Phi_V(\omega_1+S,\omega_2-S) =& \frac{\sqrt{2}\Gamma}{2\pi}\frac{1}{\omega_1+\omega_2-\omega_0+i\Gamma} \\ & \times \frac{1}{\omega_2-S-\omega_{V_2}+i\Gamma/2} \\ =& \frac{\sqrt{2}\Gamma}{2\pi}\frac{1}{\omega_1+\omega_2-\omega_0+i\Gamma} \\ & \times \frac{1}{\omega_2-\omega_{H_2}+i\Gamma/2} \\ =& \Phi_H(\omega_1,\omega_2), \end{aligned} \tag{7}$$

the spectral functions given in Eq. (6) satisfy the requirement in Eq. (4). So after $U(-S, S)$ is applied, the photon pair becomes maximally entangled.

Equivalently, our method can also be understood as a polarization-dependent spatial phase modulation scheme. In the moving reference frame in which the photons are at rest, the radiation field can be regarded as a frozen wave train. If we transform Eq. (2) from frequency space to position space, the state of the field can be rewritten as

$$\begin{aligned} |\Psi\rangle =& \frac{\sqrt{2}\Gamma}{c}\iint_{0>x_1>x_2}\frac{1}{\sqrt{2}}\left(|H_1H_2\rangle + e^{iS(x_1-x_2)/(\hbar c)}|V_1V_2\rangle\right) \\ & \times e^{\frac{\Gamma}{2}(x_2+x_1)/c}e^{i(\omega_{H_1}x_1+\omega_{H_2}x_2)/c}|x_1,x_2\rangle dx_1dx_2, \end{aligned} \tag{8}$$

where x_1 and x_2 refer to the position of the first photon and second photon, respectively. Eq. (8) is equivalent to the result given in Ref. [12].

Obviously, if we take the polarization-dependent spatial phase modulation U as

$$\begin{aligned} & U|H_1H_2;x_1,x_2\rangle = |H_1H_2;x_1,x_2\rangle; \\ & U|V_1V_2;x_1,x_2\rangle = e^{-iS(x_1-x_2)/(\hbar c)}|V_1V_2;x_1,x_2\rangle, \end{aligned} \tag{9}$$

the polarization space and position space are completely separated. To realize the second line of the transformation above, we simply take the following separate operation for each photon

$$\begin{aligned} |V_1,x_1\rangle &\to e^{-iSx_1/(\hbar c)}|V_1,x_1\rangle \\ |V_2,x_2\rangle &\to e^{iSx_2/(\hbar c)}|V_2,x_2\rangle \end{aligned} \tag{10}$$

This is equivalent to applying the following separate phase modulation to vertical polarized mode for each photon

$$\varphi_1(x_1) = -\frac{Sx_1}{\hbar c} \qquad \varphi_2(x_2) = \frac{Sx_2}{\hbar c}, \tag{11}$$

By noting that the photons are moving with a constant velocity c, the spatial phase modulation in Eq. (11) is equivalent to linearly changing the compensating phase at a fixed point as

$$\varphi_1(t) = St/\hbar \qquad \varphi_2(t) = -St/\hbar. \tag{12}$$

This is just the frequency shift by $-S$ and S, respectively, to each photon.

Realization— The frequency shift (spatial phase modulation) can be done by using two polarization-dependent phase modulators for the first photon and second photon, respectively, as shown in Fig. 2. Similar phase modulation has been used in laser spectroscopy, such as Pound-Drever-Hall laser frequency stabilization[21]. The modulator for the first (second) photon starts to linearly increase (decrease) the compensating phase (Fig. 3) before the first (second) photon arrives. The later the first (second) photon arrives at modulator 1 (2), the higher (lower) the compensating phase is applied. If compensating phases are modulated according to Eq. (12), the random phase in Eq. (8) can be completely removed, provided that the duration of phase modulation covers the duration of the field radiation.

Technically, such phase modulation can be accomplished by a commercially available optical device, such as a Pockels cell, which introduces a phase shift to vertically polarized

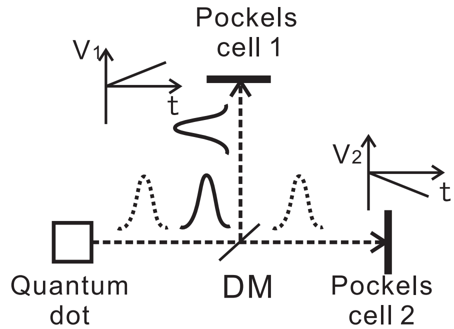

FIG. 2: Experimental set-up for our proposal. The first photon and second photon are separated by a dichroic mirror (DM). The two Pockels cells start to run before the photons arrive. They make reverse phase modulation.

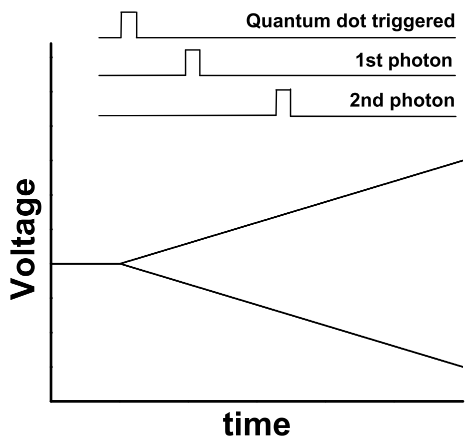

FIG. 3: Scan voltage v.s. time. Compensating phase starts to increase (decrease) for photon 1 (2) according to Eq. (12).

mode. The phase shift is proportional to the scan voltage $V(t)$, i.e. $\varphi(t) = \alpha V(t)$, where α is the phase sensitivity of the Pockels cell. By linearly changing $V(t)$, Eq. (12) can be easily satisfied. In particular, the relationship between voltage change rate dV/dt and frequency split S reads

$$\frac{dV}{dt} = \frac{S}{\hbar\alpha}. \tag{13}$$

As far as we have known, the phase sensitivity α of commercially available Pockels cells can be up to $52\ mrad/volt$ @ $830nm$[22]. This means that a rising rate at $30\ V/ns$ is required for removing an FSS of $1\ \mu eV$. To compensate larger FSS, we can arrange several Pockels cells in series along one photon's path. Furthermore, because the duration of the field radiation is about several nanoseconds[13], the scan voltage needs only last several nanoseconds. Therefore, the maximal voltage requested is only a few hundred volts according to Eq. (13), and this is easily accessible.

Discussion— Our method is not only useful in cascading decay process in quantum dot system, but also useful in many

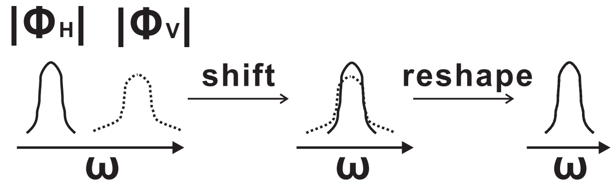

FIG. 4: Operation to maximize the overlap of $|\Phi_H|$ and $|\Phi_H|$. In the right figure, $|\Phi_H|$ and $|\Phi_H|$ overlap perfectly.

cases which need to increase the frequency indistinguishability, such as parametric-down-conversion process. Consider two spectral functions $\Phi_H = |\Phi_H|e^{i\varphi_H}$ and $\Phi_V = |\Phi_V|e^{i\varphi_V}$, where Φ_H, Φ_V, φ_H, and φ_V are functions of frequency. The larger $|\Phi_H|$ and $|\Phi_V|$ overlap, the higher the indistinguishability is. As shown in Fig. 4, by applying our frequency shift method, the overlap can be significantly increased. After that, we may further increase the overlap by changing the shape of the two spectral functions through frequency shift: Replacing Δ_1 and Δ_2 by certain appropriate functions of $\Delta_1(\omega_1)$ and $\Delta_2(\omega_2)$ in Eq. (1), $|\Phi_H|$ and $|\Phi_V|$ will be reshaped into (almost) the same form. Meanwhile, the frequency dependent terms $e^{i\varphi_H}$ and $e^{i\varphi_V}$ can be removed by the method given in Ref. [23]. So in general, after this three-step operation, one may obtain (almost) perfect indistinguishability. (Fortunately, for the spectral functions given in Eq. (6), the frequency shift alone is enough.)

Very recently, Guo's group presented an experimental scheme to improve the quality of entangled photon pairs generated by a quantum dot with FSS[23]. Here, we would like to compare our work with theirs. Although both works aim at improving the quality of entangled photon pairs, they are different in the main idea, the result, the method, and the realization. In particular, we use the frequency shift method and as a result, we can in principle obtain perfect entangled photon pairs given the spectral functions of Eq. (6) which satisfy Eq. (4). In their work[23], there is no frequency shift. Therefore, it is in principle not possible to obtain perfect overlap of two spectral functions if they initially have different frequency centers. Explicitly, their scheme is to change $e^{i\varphi_H}$ and $e^{i\varphi_V}$ in the spectral functions $|\Phi_H|e^{i\varphi_H}$ and $|\Phi_V|e^{i\varphi_V}$. Hence, through their scheme, the maximum achievable overlap of two spectral functions can never exceed the amount of overlap of $|\Phi_H|$ and $|\Phi_V|$, while our scheme can produce perfect overlap between any two spectral functions if they satisfy Eq. (4). As was just discussed above, in cases that Eq. (4) is not satisfied, our method can still help to obtain almost perfect indistinguishability if combined with some further manipulations.

In conclusion, we have proposed an experimental scheme to purify the entanglement of photon-pairs produced by a semiconductor quantum dot. The entanglement distortion caused by FSS can, in principle, be fully corrected. Since post-selection techniques are not required, our method does not decrease the photon pair collection efficiency.

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